LINEAR ALGEBRA HOMEWORK

JULY 26, 2023

In today's homework, F is a field and V is a subspace of some F^k. For $n \in \mathbb{N}$ and $1 \leq i \leq n$, denote e_i the vector that does not vanish only at row i and is 1 at row i.

Exercise 1. *Let $f : F^n \longrightarrow V$ be a linear map. Show that $f(e_1), \ldots, f(e_n) \in V$ are independent if and only if f is injective.*

Exercise 2. *Suppose that $v_1, \ldots, v_n \in V$ are independent and they span V. Show that there exists a **unique** linear bijection $f : F^n \longrightarrow V$ such that $f(e_i) = v_i$, $i = 1, \ldots, n$. (It is good to write your proof within 5 lines)*